PSEUDO-NONLINEAR MODAL ANALYSIS OF COUPLED SHEAR WALLS

by

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ABSTRACT

In the design of coupled shear walls, the coupling beams will generally be made as strong as possible. In analysis, therefore, the strength is normally known and the ductility demand is the required quantity. The substitute structure method of Shibata and Sozen is adapted to deal with this case. Limitations are discussed and analytical tests are made to show that the method provides an accuracy within the scatter due to the differences in earthquake records.

INTRODUCTION

Seismic analysis may be executed at varying levels of cost and sophistication:

1. For small or regular structures a quasi-static analysis using the equivalent forces defined by a building code is an appropriate procedure.
2. For medium size structures (e.g. buildings in the 10 to 30 storey range) an elastic modal analysis based on a design spectrum is often used. The root-sum-square of the modal forces from this analysis is divided by the available ductility associated with the particular structural system to give the yield level forces for which the building should be designed.
3. For larger or more complex structures an inelastic analysis in space and time based on appropriate earthquake records and structural characteristics is sometimes applied.

In the case of residential buildings comprised of coupled shear-walls, procedures 1 and 2 are not really appropriate. In these structures the coupling beams are generally slabs or short lintels of minimum dimensions which are difficult to reinforce in the manner suggested by Paulay[1] to give the optimum levels of yield moment and ductility. Instead, one must detail the members to give the maximum possible shear capacity, and appropriate resisting moment; the purpose of the analysis is then to determine whether the ductility demand can be met.

If this proves to be impossible, some change must be made in the structural layout, or it must be decided to accept a degree of damage in the lintels.

The load carrying mechanism in coupled walls is a combination of cantilever resisting moments in the individual walls, and vertical forces in the walls providing a resisting couple, accompanied by shears (and moments) in the coupling beams. The designer may wish to adjust the balance between these two mechanisms, by varying the yield strength of the coupling beams.

In frame structures, the available ductility is known, and the corresponding yield strength is the sought-for quantity. A linear elastic spectral analysis as described under (2) above gives the desired result; if the structure is regular, and uniform, the quasi-static analysis of (1) above will suffice. But in coupled shear walls, by contrast, the maximum available strength of the coupling beams is known or selected (to give the desired balance of resisting mechanisms), and the ductility demand is the sought-for quantity: a fundamentally different problem.

The inelastic analysis in space and time of (3) above is, of course, the best procedure. However, for small to medium structures, the cost in time and money of such analyses in both the preliminary and final design stages is prohibitive. It is the purpose of this paper to describe an economical method of analysis for this important class of structures which corresponds to spectral analysis for frame structures.

THE SUBSTITUTE STRUCTURE METHOD

The proposed method is based on a design procedure for frame structures developed by Shibata and Sozen[2], and their procedure will first be briefly described.

Fig. 1 shows a force-displacement diagram for a structural member. The ordinate is some measure of applied load (end moment, end shear, etc.) and the abscissa is some measure of displacement (end rotation, displacement, etc.). k is the initial elastic stiffness, points A represent different possible values of the yield strength, and the line AB represent lines of post-yield response. Suppose that the designer has decided in advance on an acceptable value of ductility demand, defined by the ratio of the maximum displacement, corresponding to points B, to yield displacement, corresponding to points A. It will be seen that, for constant values of ductility demand, the points B lie on a straight line of slope k/μ, where μ is defined as the "damage ratio". When the member is elastic-perfectly plastic (with no strain hardening), the damage ratio and the ductility are numerically equal.

Now, when seismic design is approached for a concrete structure, the initial stiffness k has usually been fixed by other considerations; the ductility demand which should be accepted is known in terms of the materials and detailing. The object of design is to select an appropriate yield value for the member. If the analysis were carried out on a "substitute member" of elastic stiffness k/μ, it would bring one to the point B and hence lead to the correct value of A, except that the energy absorbed would be incorrect (the area under OB being less than that under OAB). Shibata and Sozen proposed that this be corrected by use of a "substitute damping" given by

$$[1] \quad \beta = 0.02 + 0.2\left(1 - \frac{1}{\sqrt{\mu}}\right)$$

This gives a fictitious viscous damping to represent the dissipation of hysteretic energy by the real member following the path OAB. It is based on work of Gulkan and Sozen[3].

The available ductility in concrete structures varies, between columns and beams for example, and this leads to different values of substitute damping for different members. Shibata and Sozen proposed that an average or "smeared" damping ratio be computed for the r^{th} mode as

$$[2] \quad \beta_r = \frac{\sum_i (P_i^r \beta_{si})}{\sum_i P_i^r}$$

where β_{si} is the substitute damping for the i^{th} member

P_i^r is the energy of deformation of the i^{th} member, computed from

$$[3] \quad P_i^r = \frac{L_i}{6(EI)_{si}} \left\lfloor (M_{ai}^r)^2 + (M_{bi}^r)^2 - M_{ai}^r M_{bi}^r \right\rfloor$$

where L_i is the length of the i^{th} member

M_{ai}^r, M_{bi}^r are the moments at the ends of member i in the r^{th} mode

$(EI)_{si}$ is the cracked rigidity of member i.

A modal analysis of the substitute structure is made with some arbitary value of the damping, and the end moments are determined. The smeared substitute damping is computed from eqs. [1] to [3], and the forces are recalculated, with new spectral acceleration values. Shibata and Sozen suggest that

$$[4] \quad \frac{\text{Response acceleration for } \beta}{\text{Response acceleration for } \beta = 0.02} = \frac{8}{6 + 100\beta}$$

Thus, with given initial stiffness and ductility demand values, maximum forces and displacements are obtained, allowing the designer to select reinforcement so as to give the implied yield moments.

These are the essential features of the substitute structure method, used by Shibata and Sozen to determine the required yield values for frame structures in which the initial stiffness and available ductility are known in advance.

PROPOSED METHOD

The substitute structure method was previously modified by Yoshida et al[4] for use in analysing existing buildings for retro-fit purposes. The problem there is similar to that described above for design of coupled shear-walls: the member capacities and stiffnesses are known in advance and the ductility demand is the sought-for quantity. The present investigation was designed to test the accuracy of the modified procedure when used to analyse reasonably regular coupled shear-wall systems.

The suggested modification is an iterative procedure which converges onto the correct yield values, damage ratios, and hence ductilities. The steps are as follows:

1. An elastic modal analysis is made of the structure, using, in the first iteration, the initial stiffnesses and appropriate damping values. The root-sum-square forces are calculated.
2. In subsequent iterations, the analysis is repeated, but those members, whose end moments exceed their yield values, have their stiffnesses reduced to

   $$k_{n+1} = k_n \,/\, \mu_{n+1}$$

   where

   $$\mu_{n+1} = \mu_n \frac{M_n}{M_y} > 1$$

   k_n = stiffness used in n^{th} iteration

   k_{n+1} = stiffness used in $n+1^{th}$ iteration

   μ_n = damage ratio used in n^{th} iteration

   ($\mu_1 = 1$, or an estimated value may be used)

   μ_{n+1} = damage ratio for the $n+1^{th}$ iteration

   M_n = the larger root-sum-square end moment from the n^{th} iteration

   M_y = the yield moment

   For each iteration after the first a smeared damping value for each mode is calculated as described above for the original substitute structure method.

3. When all the member forces are either below or within a tolerable limit of their yield values, the analysis is halted. The damage

ratios, or the ductility demands implied thereby, are the required values.

It was found that a twofold convergence criterion gave the best results: all maximum end moments should be less than or within 5% of the moment capacity of the member; and a limit should be placed on the change in damage ratio from the penultimate iteration to the last. This limit was set at 1% of the last damage ratio (if that was greater than 5) or at an absolute value of 0.1 if the final damage ratio was less than 5. A convergence speeding procedure was found useful: when there was a monotonic trend in any damage ratio, it was overcorrected somewhat for the next iteration.

LIMITATIONS OF PROPOSED METHOD

In their original paper Shibata and Sozen restricted their method to structures

1. which could be subjected to planar analyses.
2. which had no abrupt changes in geometry or mass over their height.
3. in which the damage ratios were the same in the columns on any one vertical axis and in the beams of any one bay.
4. in which the members and joints were detailed to prevent significant strength decay with repeated load reversals.
5. in which the dynamic response was governed by the structural rather than the architectural features.

They also required that the spectral acceleration should not increase with increases in the fundamental period of the structure. These are not undue restrictions for the proposed analysis of coupled shear-walls; our studies indicate that violation of the third does not lead to serious error.

In the present work, further common simplifications are introduced:
Beams and columns are modelled as line members;
Walls are modelled as line members with rigid segments within the joint area;
P - delta effects are omitted;
Masses are lumped at nodes, one mass per floor;
Floors are assumed to act as diaphragms rigid in their own planes;
Members must be symmetric since damage ratios are based only on the largest end moment and no differentiation is made between positive and negative bending moment;
Changing axial and shear forces are not considered in determining the yield state of the members, although initial gravity forces are included;
Account is taken of axial shortening generated from earthquake forces, but the static forces generated by gravity loads are not included in determination of damage ratios;
These simplifications, common in structural analysis, are not necessary for the present purpose, and can be removed to the extent that further computational effort is felt to be necessary.

TESTS OF PROPOSED METHOD

A number of test structures was analysed by the proposed method, and the results were compared with those obtained from a non-linear time-step analysis, with the latter assumed to represent the truth. For this purpose, the program DRAIN-2D was used; in it, an extended version of the Takeda model is used to represent the degradation of concrete stiffness taking account of the influence of axial force on yield. Structures were represented by line members with rigid joint areas as described above; nodes were placed on the neutral axes of the uncracked members, although it is recognized that this position would not really be fixed during dynamic response. Viscous damping was included at 2% of critical to represent the effect of non-structural components, since hysteretic damping of the structure itself is, of course, included automatically in the non-linear analysis. Some investigators have used much higher values of viscous damping in this context, but, if this is felt to be necessary, eq. [1] should be modified for comparison with the proposed method. (Note that the first term in eq. [1] represents the damping when $\mu = 1$: when the member is still elastic). All test structures were assumed to have fixed bases, repesenting the typical case where walls terminate in the larger walls of the basement parking structure. Otherwise member sizes and loads were chosen to be reasonably representative of practical cases.

The same test earthquake records were adopted, with scaled peak accelerations, as had been used by Shibata and Sozen in their original work (see Table I). In the proposed method, the spectrum A, (see Fig.3) which was developed by Shibata and Sozen from these records was used. The definition of member ductility used for comparisons purposes was the maximum absolute value of the angle shown in Fig. 2 divided by its yield value.

Test Structures: The layout of the test structures is shown in Fig. 4. Five storey coupled walls were studied, with coupling beam capacities of 60 Kips-ft and 100 Kips-ft; peak ground acceleration was set at 20% and 50% of gravity, and the masses were varied by a factor of 4 to examine the effect of period change. Also a ten storey coupled wall and a 16 storey coupled wall connected to an extra uncoupled wall similar to the structure discussed by Fintel and Ghosh[5] were tested. In the latter case, 2% viscous damping was used, rather than 10%, as discussed above. Furthermore, it was found that, at the high ductility demands occurring in this structure, the 5% strain hardening used by Fintel and Ghosh led to ultimate moments equal to about twice the yield moments. To be consistent with the program written for the proposed method, based on elastic-perfectly plastic response, the strain hardening was reduced to 0.5%. These changes are not, of course, necessary for the method; they were only required to be consistent with the parameters previously adopted for the present study. This structure had a stiffness change at midheight, as well as variations in the masses, so that it provided a severe test of the method.

Results of Comparisons: Typical results for the 5-storey structures are shown in Figs. 5 to 8. The proposed method showed the correct distribution of damage ratios, but was excessively conservative at lower periods. This may be due to spectrum A being a rather conservative envelope at this period. At higher periods the method worked well, although it is clear that the true response varies widely and unpredictably with earthquake input, and that great accuracy is not attainable by any modal method.

Results of the 10 storey walls are shown on Figs. 9 and 10. At these periods (see Fig. 3) use of spectrum A with the proposed method yielded a close upper bound on the response, with an accurate prediction of the form of the damage ratio distribution.

In the case of the 16-storey structure, Figs. 11 to 13 show that the procedure again gave good qualitative results, with reasonable quantitative accuracy. The dependence of the actual detailed response on earthquake input is again evident.

CONCLUSIONS

It is felt that the proposed procedure provides a method of analysing coupled shear walls which is equivalent to the usual elastic modal analysis of frame structures. The accuracy is probably about the same, and well within the spread of actual results with variations in earthquake input. The objective is somewhat different, however: to check a given layout, in which the "columns" (walls) respond in a way entirely different from the coupling beams, and in which the widely varying ductility demands or damage ratios are to be determined. The acceptability of the calculated response in a given case is a matter for the designer, and is beyond the scope of the present discussion, but a word of warning will be inserted here: the coupling beams will undergo far more excursions beyond their yield limits than is generally the case with frame members, and the response to high reversing shear forces can well cause a reduction in the available bending ductility. But, as noted in the introduction, if the ductility demands are felt to be excessive, the behaviour of the system must be changed by altering the layout, or deciding to accept damage in the coupling beams.

ACKNOWLEDGEMENTS

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REFERENCES

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EARTHQUAKE	DATE	AMAX	RECORDING STATION
EL CENTRO (NS)	MAY 18, 1949	0.348	EL CENTRO SITE IMPERIAL VALLEY IRRIGATION DISTRICT
EL CENTRO (EW)	MAY 18, 1940	0.182	EL CENTRO SITE IMPERIAL VALLEY IRRIGATION DISTRICT
KERN COUNTY (S69E)	JULY 21, 1952	0.179	TAFT LINCOLN SCHOOL TUNNEL
KERN COUNTY (n21E)	JULY 21, 1952	0.156	TAFT LINCOLN SCHOOL TUNNEL

NOTE: AMAX = MAXIMUM ACCELERATION OF ORIGINAL RECORD DURING SEGMENT OF RECORD USED.

FIRST TEN SECONDS OF EACH RECORD USED.

EARTHQUAKE RECORDS USED IN DRAIN-2D COMPUTER RUNS.

TABLE 1

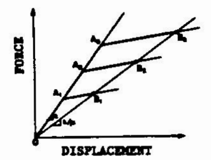

FIG. 1 MEMBER FORCE-DISPLACEMENT DIAGRAM

FIG. 2 ANGLE USED FOR CALCULATION OF MEMBER DUCTILITY

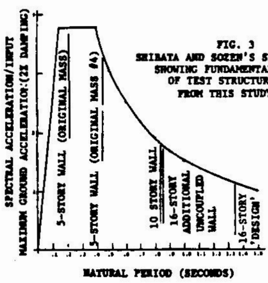

FIG. 3 SHIBATA AND SOZEN'S SPECTRUM 'A' SHOWING FUNDAMENTAL PERIOD OF TEST STRUCTURES FROM THIS STUDY

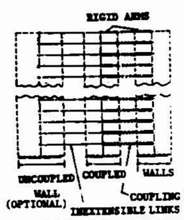

FIG. 4 GENERAL TEST STRUCTURE CONFIGURATION

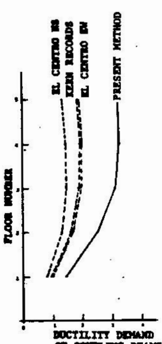

FIG. 5 5-STORY WALL

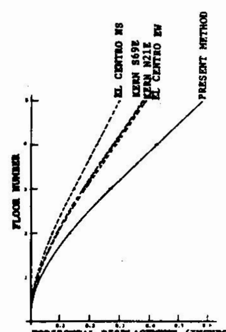

FIG. 6 DISPLACEMENT ENVELOPES FOR THE 5-STORY WALL

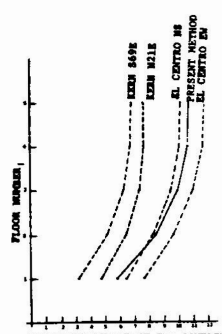

FIG. 7 5-STORY WALL (MASS=4 TIMES ORIGINAL RUN)

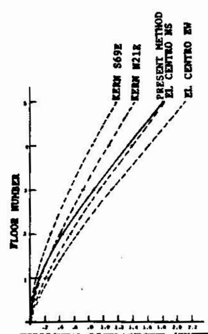

FIG. 8 DISPLACEMENT ENVELOPES FOR THE 5-STORY WALL (MASS=4 TIMES ORIGINAL RUN)

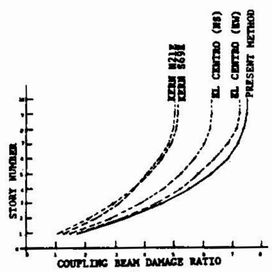

FIG. 9 TEN STORY WALL.

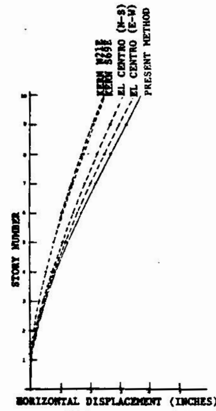

FIG. 10 DEFLECTION ENVELOPES FOR THE 10-STORY WALL

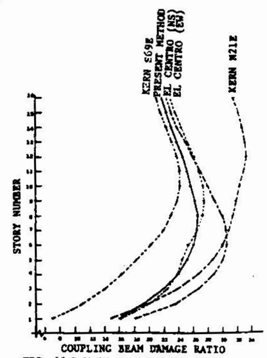

FIG. 11 DAMAGE RATIOS FOR THE 16-STORY COUPLED WALL WITH ATTACHED UNCOUPLED WALL

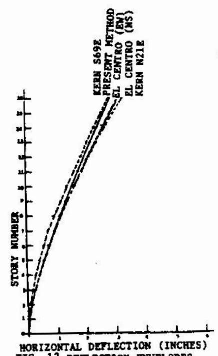

FIG. 12 DEFLECTION ENVELOPES FOR THE 16-STORY COUPLED WALL WITH ATTACHED UNCOUPLED WALL

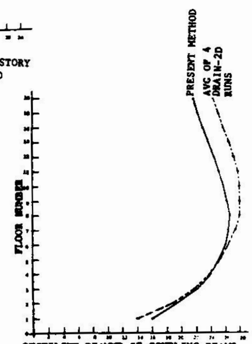

FIG. 13 AVERAGE DUCTILITY DEMAND OF COUPLING BEAMS FOR THE 16-STORY COUPLED WALL WITH ATTACHED UNCOUPLED WALL